Student Number ....

Barker College

2007
TRIAL HIGHER SCHOOL CERTIFICATE

Mathematics Extension 1

Staff Involved:

PM FRIDAY 10 AUGUST

- **MRB***
- **JGD***
- **VAB**
- **PJR**
- **GDH**
- **RMH**
- **BHC**

75 copies

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Make sure your Barker Student Number is on ALL pages
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- ALL necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1–7
- All questions are of equal value

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Attempt Questions 1–7
ALL questions are of equal value

Answer each question on a SEPARATE sheet of paper

Marks

Question 1 (12 marks) **[START A NEW PAGE]**

(a) Evaluate $\displaystyle\int_0^2 \frac{dx}{\sqrt{4 + x^2}}$ **2**

(b) Using the substitution $u = x^4 + 8$, or otherwise, find $\displaystyle\int \frac{4x^3 dx}{\sqrt{x^4 + 8}}$ **2**

(c) Evaluate $\displaystyle\lim_{a \to 0} \frac{\sin 5a}{\tan \frac{a}{2}}$ **2**

(d) (i) Write down the expanded form of $\cos(A + B)$ **1**

(ii) Hence, find the exact value of $\cos 75^o$ **2**

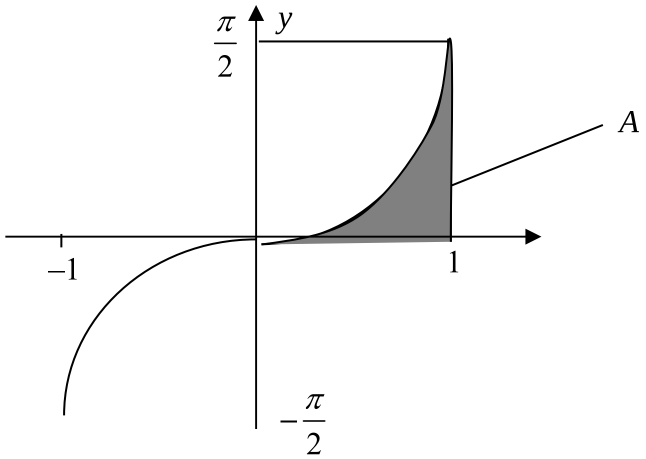
(e) For what values of c is the line $y = 3x + c$ tangent to $y = x^3$ **3**

Marks

Question 2 (12 marks) **[START A NEW PAGE]**

(a) Solve $\dfrac{4}{5 - x} \geq 1$ **3**

(b) A represents the area bounded by the x-axis, $y = \sin^{-1} x$ and the line $x = 1$.



(i) Write down two different expressions (without evaluating them) involving integrals that can be used to find the magnitude of A. **2**

(ii) Find the value of A. **2**

(c) Let $f(x) = 2\cos^{-1}(x - 1)$

(i) State the domain and range of the function $f(x)$ **2**

(ii) Find the gradient of the graph of $y = f(x)$ at the point where $x = \dfrac{1}{2}$ **2**

(iii) Sketch the graph of $y = f(x)$ **1**

Marks

Question 3 (12 marks) **[START A NEW PAGE]**

(a) Erin has loaded 5 mp3 song tracks onto her mobile phone.

The songs are played at random.

She can listen to the songs in sets, with one to five songs in each set,

(i.e. the songs can be played in sets of 1, 2, 3, 4 or 5 songs).

(i) How many different ordered sets of 3 songs can she hear? **1**

(ii) How many different ordered sets of songs can she hear in total? **2**

(b) John and Mike decide to play a game.

They take it in turns to toss two coins, the first to throw two heads wins the game.

Mike is the first to toss the two coins.

(i) What is the probability that Mike wins on his first throw? **1**

(ii) What is the probability that John wins on his first throw? **1**

(iii) What is the probability that Mike wins on his second throw,
(i.e. the third throw of the game)? **2**

(iv) What is the probability of Mike **eventually** winning the game?
(You may wish to draw a tree diagram). **2**

(c) Find the equation of the curve which passes through the point $\left(\frac{\pi}{4}, \frac{\pi}{8}\right)$

and has a gradient function of $f'(x) = \sin^2 x$ **3**

Marks

Question 4 (12 marks) **[START A NEW PAGE]**

(a) (i) Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \beta)$ where $0 < \beta < \dfrac{\pi}{2}$ **2**

(ii) A particle moves such that its displacement is given by $\ddot{x} = -25x$

Show that $x = 3\cos 5t + 4\sin 5t$ is a solution of this differential equation. **1**

(iii) Find the maximum displacement of this particle. **1**

(b) A function is defined as $f(x) = 10x^2 - x^4 - 9$

(i) Find all stationary points and determine their nature. **2**

(ii) Sketch this function. **2**

(c) A particle moves on a straight line such that its distance from the origin at time t seconds is x metres.

(i) Prove that $\dfrac{d^2x}{dt^2} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$ where v is the velocity of the particle. **1**

(ii) If $\dfrac{d^2x}{dt^2} = 10x - 2x^3$ and $v = 0$ at $x = 1$, find v in terms of x. **2**

(iii) Using part (b), or otherwise, determine the maximum velocity of this particle. **1**

Marks

Question 5 (12 marks) **[START A NEW PAGE]**

(a) (i) Prove, by mathematical induction for n an integer, where $n \geq 1$

$$\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2} \text{..........} - \frac{1}{x^n} = \frac{1}{x^n(x-1)}$$ **3**

(ii) Hence, or otherwise, find the exact value of

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} \text{...........} + \frac{1}{2^{10}}$$ **1**

(b) Consider the binomial expansion of $\left(x^2 + \frac{2}{x}\right)^{10}$

(i) Find the coefficient of x^2 **2**

(ii) Is there a constant term for this expansion? Explain your answer. **2**

(c) When the temperature of a body is T degrees **above** the temperature of its surroundings, its temperature falls at a rate proportional to T,

i.e. $\frac{dT}{dt} = -kT$ where k is constant and t denotes time.

(i) Show that $T = Ae^{-kt}$ satisfies this equation where A is constant. **1**

(ii) If the body is initially at 95°C, the surroundings at 25°C, and after 15 minutes it cools to 75°C, find the value of the constants A and k. **2**

(iii) What will be the temperature of the body after a further 20 minutes to the nearest degree? **1**

Marks

Question 6 (12 marks) **[START A NEW PAGE]**

(a) Write down the general solution of $\cos 2\theta = \sin\theta$ **3**

(b) P is the point $\left(2ap\ ,\ ap^2\right)$ on the parabola $x^2 = 4ay$ and l is the tangent to this parabola at P.

(i) Prove that the equation of ℓ is $y = px - ap^2$ **2**

(ii) If ℓ cuts the x-axis at A and the y-axis at B, find the coordinates of A and B. **2**

(iii) In what ratio does A divide the interval BP? **2**

(iv) What is the gradient of the line joining P to the focus S of the parabola? **1**

(v) Show that ℓ makes equal angles with the y-axis and with PS. **2**

Marks

Question 7 (12 marks) **[START A NEW PAGE]**

(a) Pete and Graham are both standing 50 metres apart on level ground on No. 1 Oval. Pete throws a ball from a height of 1.9 metres which Graham catches two seconds later (without it bouncing), also at a height of 1.9 metres.

You may assume:

I there is no air resistance and the value of g is 10 (ms^{-2}).

II the equations of motion, are:

$$\dot{x} = V\cos\alpha \qquad \dot{y} = -10t + V\sin\alpha$$

$$x = Vt\cos\alpha \qquad y = -5t^2 + Vt\sin\alpha + 1\cdot 9$$

where V is the initial velocity, α is the angle of projection, t is the time taken and the origin is at Pete's feet.

(i) Find the initial velocity and the angle of projection. **3**

(ii) Find the maximum height of the ball above the ground. **2**

(iii) Pete throws another ball with the same initial velocity and from the same starting height (1.9 metres above the level ground), but he wants to **maximise** the distance he throws horizontally. **2**
How far away should Graham **now** stand in order to catch this second throw (without bouncing and at a height of 1.9 metres)?

(b) A series of lines are drawn on a Cartesian diagram such that $y = nx$, where n can be one of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Triangles are formed using two of the above lines and the line $x = 1$.

(i) If $n < 4$, how many such triangles can be formed? **1**

(ii) How many triangles can be formed for $n < 10$? **1**

(iii) (α) How many different **pairs** of triangles can be formed for $n < 10$? **1**

(β) How many of these **pairs** of triangles have no common area? **2**

End of Paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\quad x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a > 0,\quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\quad x > 0$

2007 Mathematics Extension 1 Trial HSC - 10/8/07 Solutions

Question 1

(a) $\int_0^2 \frac{dx}{\sqrt{4+x^2}} = \left[\ln\left(x+\sqrt{x^2+4}\right)\right]_0^2$

$= \ln\left(2+\sqrt{4+4}\right) - \ln\left(0+\sqrt{0+4}\right)$

$= \ln(2+\sqrt{8}) - \ln(\sqrt{4})$

$= \ln\left(\frac{2+2\sqrt{2}}{2}\right)$

$= \ln(1+\sqrt{2})$

(b) $u = x^4+8$

$\frac{du}{dx} = 4x^3$

$\therefore du = 4x^3\,dx$

$\int \frac{4x^3\,dx}{\sqrt{x^4+8}} = \int \frac{du}{\sqrt{u}}$

$= \int u^{-1/2}\,du$

$= \frac{u^{1/2}}{1/2} + C$

$= 2\sqrt{x^4+8} + C$

(c) $\lim_{a\to 0} \frac{\sin 5a}{\tan\frac{a}{2}}$

$= \lim_{a\to 0} \frac{\sin 5a}{5a} \times \frac{a/2}{\tan\frac{a}{2}} \times \frac{5}{1/2}$

$= 1 \times 1 \times 10$

$= 10$

(d)(i) $\cos(A+B)$

$= \cos A\cos B - \sin A\sin B$

(ii) $\cos 75° = \cos(30°+45°)$

$= \cos 30°\cos 45° - \sin 30°\sin 45°$

$= \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}} - \frac{1}{2}\times\frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$

(e) $y = 3x + c \quad \therefore m = 3$

$y = x^3$

$\frac{dy}{dx} = 3x^2$

$\therefore 3x^2 = 3$

$\therefore x = 1$ or -1, $y = 1$ or -1 $\therefore$ pts $(1,1)$ and $(-1,-1)$

subst. $(1,1)$ into $y = 3x + C$

$\therefore 1 = 3 + C \quad \therefore C = -2$

subst. $(-1,-1)$ into $y = 3x + C$

$\therefore -1 = -3 + C \quad \therefore C = 2$

$\therefore C = \pm 2$

Question 2

(a) $(5-x)^2 \times \frac{4}{(5-x)} \geq 1 \times (5-x)^2$

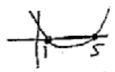
$\therefore 4(5-x) \geq (5-x)^2$

$\therefore 0 \geq (5-x)^2 - 4(5-x)$

$\therefore 0 \geq (5-x)[5-x-4]$

$\therefore 0 \geq (5-x)(1-x)$

$\therefore (5-x)(1-x) \leq 0$



But $x \neq 5$

$\therefore 1 \leq x < 5$

(b)(i) $y = \sin^{-1} x$

$\therefore x = \sin y$

$A = \frac{\pi}{2}\times 1 - \int_0^{\pi/2} \sin y\,dy$

OR

$A = \int_0^1 \sin^{-1} x\,dx$

(ii) $A = \frac{\pi}{2} - \int_0^{\pi/2} \sin y\,dy$

$= \frac{\pi}{2} - \left[-\cos y\right]_0^{\pi/2}$

$= \frac{\pi}{2} - \left[-\cos\frac{\pi}{2} + \cos 0\right]$

$= \frac{\pi}{2} - [0+1]$

$= \left(\frac{\pi}{2} - 1\right)$ units^2

(c)(i) $-1 \leq x - 1 \leq 1$

Domain: $0 \leq x \leq 2$

Range: $0 \leq y \leq 2\pi$

(ii) $f(x) = 2\cos^{-1}(x-1)$

$f'(x) = \frac{2x-1}{\sqrt{1-(x-1)^2}}$

$= \frac{-2}{\sqrt{1-(x^2-2x+1)}}$

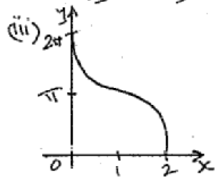
$= \frac{-2}{\sqrt{2x-x^2}}$

$f'\left(\frac{1}{2}\right) = \frac{-2}{\sqrt{1-\frac{1}{4}}}$

$= \frac{-2}{\frac{\sqrt{3}}{2}}$

$= \frac{-4}{\sqrt{3}} = \frac{-4\sqrt{3}}{3}$

(iii)



Question 3

(a)(i) $^5P_3 = 60$

(ii) Total $= {}^5P_1 + {}^5P_2 + {}^5P_3 + {}^5P_4 + {}^5P_5$

$= 5 + 20 + 60 + 120 + 120$

$= 325$

(b)(i) $\frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$

(ii) Mike & John win

$= \frac{3}{4}\times\frac{1}{4} = \frac{3}{16}$

(iii) Mike not win & John not win & Mike win

$= \frac{3}{4}\times\frac{3}{4}\times\frac{1}{4} = \frac{9}{64}$

(iv) $\frac{1}{4} + \left(\frac{3}{4}\right)^2\times\frac{1}{4} + \left(\frac{3}{4}\right)^4\times\frac{1}{4} + \ldots$

Infinite G.P. $a = \frac{1}{4}$, $r = \frac{9}{16}$

P(Mike wins eventually) $= S = \frac{\frac{1}{4}}{1-\frac{9}{16}} = \frac{\frac{1}{4}}{7/16} = \frac{4}{7}$

Q3(c)

$\cos 2x = 1 - 2\sin^2 x$

$\therefore 2\sin^2 x = 1 - \cos 2x$

$\therefore \sin^2 x = \frac{1}{2}(1-\cos 2x)$

$f'(x) = \sin^2 x$

$= \frac{1}{2}(1-\cos 2x)$

$\therefore f(x) = \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + C$

$\therefore \frac{\pi}{8} = \frac{1}{2}\left(\frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{2}\right) + C$

$\therefore \frac{\pi}{8} = \frac{\pi}{8} - \frac{1}{4}\times 1 + C$

$\therefore C = \frac{1}{4}$

$f(x) = \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + \frac{1}{4}$

$= \frac{1}{4}(2x - \sin 2x + 1)$

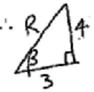
Question 4

(a)(i) $3\cos\theta + 4\sin\theta = R\cos(\theta-\beta)$

$= R\cos\theta\cos\beta + R\sin\theta\sin\beta$

$\therefore 3 = R\cos\beta \quad \therefore \cos\beta = \frac{3}{R}$

$4 = R\sin\beta \quad \sin\beta = \frac{4}{R}$



$\therefore R = 5$

$\beta = \tan^{-1}\left(\frac{4}{3}\right)$

$\therefore \beta = 0.927\ldots$

$\therefore 3\cos\theta + 4\sin\theta$

$= 5\cos(\theta - 0.927\ldots)$

(ii) $x = 3\cos 5t + 4\sin 5t$

$v = \dot{x} = -15\sin 5t + 20\cos 5t$

$a = \ddot{x} = -75\cos 5t - 100\sin 5t$

$= -25(3\cos 5t + 4\sin 5t)$

$\therefore \ddot{x} = -25x$ (ie SHM)

(iii) $x = 3\cos 5t + 4\sin 5t$

$\therefore x = 5\cos(5t - 0.927\ldots)$

since SHM, maximum displacement = 5 units

(b)(i) $f(x) = 10x^2 - x^4 - 9$

$f'(x) = 20x - 4x^3$

stat pts when $f'(x) = 0$

$\therefore 0 = 4x(5 - x^2)$

$\therefore x = 0, \sqrt{5}, -\sqrt{5}$

$y = -9, 16, 16$

$f''(x) = 20 - 12x^2$

$f''(0) = 20 > 0$ ∪

$\therefore$ Min t.p. at $(0,-9)$

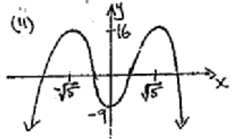
$f''(\sqrt{5}) = -40 < 0$ ∩

$\therefore$ Max t.p. at $(\sqrt{5}, 16)$

$f''(-\sqrt{5}) = -40 < 0$ ∩

$\therefore$ Max t.p. at $(-\sqrt{5}, 16)$

(ii)



(c)(i) $\frac{d^2x}{dt^2} = \frac{dv}{dt}$

$= \frac{dv}{dx}\times\frac{dx}{dt}$

$= \frac{dv}{dx}\times v$

$= \frac{dv}{dx}\times\frac{d}{dv}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

(ii) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 10x - 2x^3$

$\therefore \frac{1}{2}v^2 = 5x^2 - \frac{x^4}{2} + C$

When $x = 1$, $v = 0$

$\therefore 0 = 5 - \frac{1}{2} + C$

$\therefore C = -4\frac{1}{2}$

$\therefore \frac{1}{2}v^2 = 5x^2 - \frac{x^4}{2} - 4\frac{1}{2}$

$\therefore v^2 = 10x^2 - x^4 - 9$

$\therefore v = \pm\sqrt{10x^2 - x^4 - 9}$

(iii) From above graph

Max velocity $= +\sqrt{16} = 4$ m/s

Question 5

(a)(i) Prove true for $n = 1$

LHS $= \frac{1}{x-1} - \frac{1}{x}$ RHS $= \frac{1}{x(x-1)}$

$= \frac{x-(x-1)}{x(x-1)}$

$= \frac{1}{x(x-1)}$ $\therefore$ True for $n = 1$

Assume statement is true for $n = k$, ie.

$\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2} \ldots - \frac{1}{x^k} = \frac{1}{x^k(x-1)}$

Now prove true for $n = k+1$

ie. $\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2} - \frac{1}{x^k} - \frac{1}{x^{k+1}}$

$= \frac{1}{x^k(x-1)} - \frac{1}{x^{k+1}}$

$= \frac{1}{x^k(x-1)} - \frac{1}{x^k x^1}$

$= \frac{x^1 - (x-1)}{x^k(x-1)x^1}$

$= \frac{x-1+1}{x^k x^1(x-1)}$

$= \frac{1}{x^{k+1}(x-1)}$

$\therefore$ If statement is true for $n = k$, then it is true for $n = k+1$

Thus, if statement is true for $n = 1$, then it is true for $n = 2$

And if it is true for $n = 2$, it is true for $n = 3$, and so on

Hence, statement is true for $n \geq 1$ (where n. is an integer)

(ii) G.P. with $a = \frac{1}{2}$, $r = \frac{1}{2}$, $n = 10$

$S_{10} = \frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^{10} - 1\right)}{\frac{1}{2} - 1}$

$= \frac{\frac{1}{2}\left(\frac{1}{1024} - 1\right)}{-1/2}$

$= -\left(\frac{-1023}{1024}\right) = \frac{1023}{1024}$

(b)(i) General term is given by

${}^{10}C_r\,(x^2)^{10-r}(2x^{-1})^r$

$= {}^{10}C_r\,2^r \cdot x^{20-2r} \cdot x^{-r}$

$= {}^{10}C_r\,2^r x^{20-3r}$

Q5(b)(i) (cont.)

x^2 occurs when

$20-3r=2$

$\therefore 3r=18$

$\therefore r=6$

$\therefore$ Coeff't of x^2 is

${}^{10}C_6\, 2^6 = 13440$

(ii) Constant term $\Rightarrow x^0$

ie. $20-3r=0$

$\therefore 3r=20$

$\therefore r = 6\frac{2}{3}$

But r must be integer.

$\therefore$ No constant term

(c)(i) $T=Ae^{-kt}$

$\therefore \frac{dT}{dt} = -kAe^{-kt}$

$\therefore \frac{dT}{dt} = -kT$

(ii) When $t=0$, $T_0 = 95-25 = 70$

$\therefore 70 = Ae^0 \quad \therefore A=70$

When $t=15$ $T=75-25 = 50$

$\therefore 50 = 70e^{-15k}$

$\therefore \frac{5}{7} = e^{-15k}$

$\therefore -15k = \log_e\left(\frac{5}{7}\right)$

$\therefore k = -\frac{1}{15}\log_e\left(\frac{5}{7}\right) = 0.022431\ldots$

(iii) $T = 70e^{-35\times -\frac{1}{15}\ln\left(\frac{5}{7}\right)}$

$= 31.925\ldots$

$\doteqdot 32°C$

$\therefore$ Temperature of body is $32+25 = 57°C$

Question 6

(a) $\cos 2\theta = \sin\theta$

$\therefore 1-2\sin^2\theta = \sin\theta$

$\therefore 2\sin^2\theta + \sin\theta - 1 = 0$

$\therefore (2\sin\theta - 1)(\sin\theta+1) = 0$

$\therefore \sin\theta = \frac{1}{2}$ or $\sin\theta = -1$

$\therefore \theta = n\pi + (-1)^n\frac{\pi}{6}$

OR

$\theta = n\pi + (-1)^n\left(-\frac{\pi}{2}\right)$

(b)(i) $y = \frac{x^2}{4a} \quad \therefore \frac{dy}{dx} = \frac{x}{2a}$

When $x=2ap$, $\frac{dy}{dx} = \frac{2ap}{2a} = p$

$\therefore$ Equation of tangent l is

$y - ap^2 = p(x-2ap)$

$\therefore y - ap^2 = px - 2ap^2$

$\therefore y = px - ap^2$

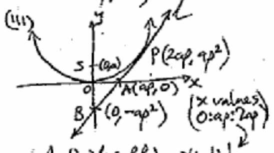
(ii) When $x=0$, $y=-ap^2$

$\therefore B(0,-ap^2)$

When $y=0$, $x = \frac{ap^2}{p} = ap$

$\therefore A(ap, 0)$

(iii)



$\therefore$ A divides BP in ratio 1:1

(iv) $S(0,a)$ $P(2ap, ap^2)$

$m_{PS} = \frac{ap^2-a}{2ap-0} = \frac{a(p^2-1)}{2ap}$

$m_{PS} = \frac{p^2-1}{2p}$

(v) $d_{SB} = SB + OS$

$= ap^2 + a$

$d_{PS} = \sqrt{(ap^2-a)^2 + (2ap-0)^2}$

$= \sqrt{a^2p^4 - 2a^2p^2 + a^2 + 4a^2p^2}$

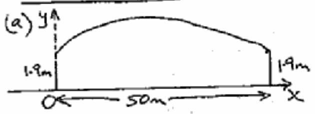
$= \sqrt{a^2p^4 + 2a^2p^2 + a^2}$

$= \sqrt{(ap^2+a)^2} = ap^2 + a = d_{SB}$

$\therefore \triangle SPB$ is ... $\therefore \hat{SBP} = \hat{SPB}$

Question 7

(a)



(i) When $t=2$, $x=50$, $y=1.9$

$50 = 2V\cos\alpha$

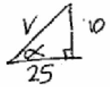
$\therefore 25 = V\cos\alpha \quad \therefore \cos\alpha = \frac{25}{V}$

Also,

$1.9 = -20 + 2V\sin\alpha + 1.9$

$\therefore 20 = 2V\sin\alpha$

$\therefore \sin\alpha = \frac{10}{V} \rightarrow \tan\alpha = \frac{\frac{10}{V}}{\frac{25}{V}}$



$\therefore \tan\alpha = \frac{10}{25}$

$\therefore \alpha = \tan^{-1}\left(\frac{2}{5}\right)$

$V = \sqrt{10^2 + 25^2}$

$= \sqrt{725} = 5\sqrt{29}$

$\therefore$ Initial velocity $= 5\sqrt{29}$ m/s ($\doteqdot 27$ m/s)

and angle of projection $\doteqdot 21°48'$

(ii) Max height @ halfway point

ie. $y=0$ or $t=1$

When $t=1$,

$y = -5\times 1 + 5\sqrt{29}\times 1 \times \frac{10}{5\sqrt{29}} + 1.9$

$= -5 + 10 + 1.9$

$= 6.9$ m

(iii) For maximum distance

$\alpha = 45°$ $V = 5\sqrt{29}$ m/s, $y = 1.9$

$\therefore 1.9 = -5t^2 + 5\sqrt{29}\,t\sin 45° + 1.9$

$\therefore 0 = -5t^2 + \frac{5t\sqrt{29}}{\sqrt{2}}$

$\therefore -5t\left(t - \sqrt{\frac{29}{2}}\right)$

$\therefore t=0$ or $t = \sqrt{14.5}$ ($\doteqdot 3.8$ sec)

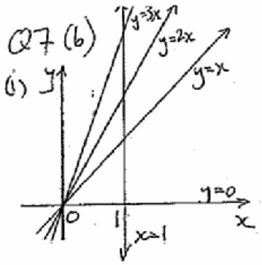
$\therefore$ Max dist $= 5\sqrt{29}\times\sqrt{\frac{29}{2}}\times\frac{1}{\sqrt{2}}$

$= \frac{5\times 29}{2}$

$= 72.5$ m

Q7 (b)

(i)



Since $x=1$ forms one side of each triangle, we need to choose any two lines from $y=0$, $y=x$, $y=2x$ and $y=3x$

ie. No. of triangles $= {}^4C_2 = 6$ (OR 3 "single" + 2 "double" + 1 "triple" = 6)

(ii) Similarly for $n \le 10$,

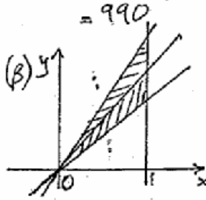
No. of triangles $= {}^{10}C_2 = 45$ (OR $9+8+7+6+5+4+3+2+1 = 45$)

(iii)(α) To choose a pair of triangles from a total of 45 triangles

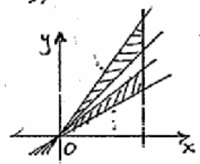
$= {}^{45}C_2$ (OR $\frac{45\times 44}{2}$)

$= 990$

(β)



To get areas adjacent, need to choose 3 lines from the 10 available, ie. ${}^{10}C_3$



To get areas with a "gap" between them, need to only choose 4 lines from the 10 available, ie. ${}^{10}C_4$

All other options are just more complex versions of these two scenarios.

$\therefore$ No. of pairs of triangles with no common area $= {}^{10}C_3 + {}^{10}C_4$

$= 120 + 210$

$= 330$